Graphics Seminar Series 2021

A presentation on paper title :

3D Skeletons: A State-of-the-Art Report

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Skeleton :

- Skeleton is a thin centered structure which jointly describes the topology and the geometry of the shape. Skeletons provide an alternative to classical boundary or volumetric representations.
- Object, or shape is a compact spatial subset $O \subset R^n$ with a 2−manifold boundary $S = \partial O$, with a focus on three-dimensional shapes ($n = 3$). In R^2 , we refer to S as the shape's contour; the medial skeleton of O is called the medial axis or 2D skeleton. In R^3 , S will be referred to as the surface of O; the corresponding medial skeleton of O is called the medial surface or surface skeleton.
- Skeletons have known many definitions. These definitions are, usually, equivalent, i.e., they imply a unique 'formal' skeleton for a given shape O.

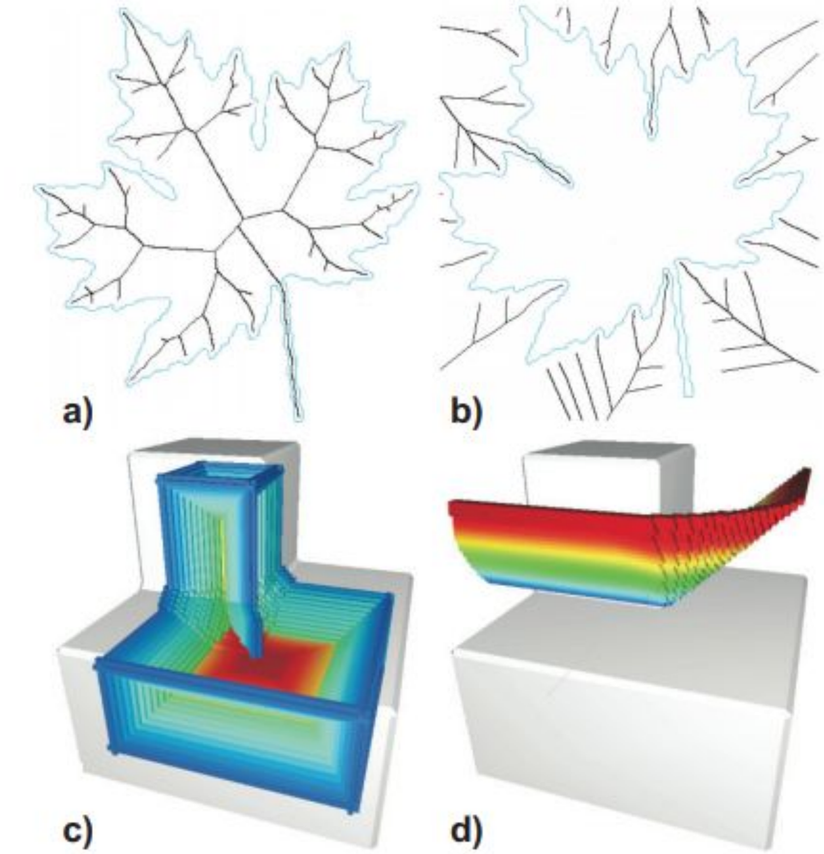

Figure 1: Skeletons (a,c) and their complements (b,d) for 2D shapes (a,b) and 3D shapes (c,d).

Medial Skeleton :

The medial skeleton of a shape knows several equivalent definitions.

1. **Maximally Inscribed Ball :** The Medial Axis Transform MAT(O) of O is the set of centers M and corresponding radius R of all maximal inscribed balls in O.

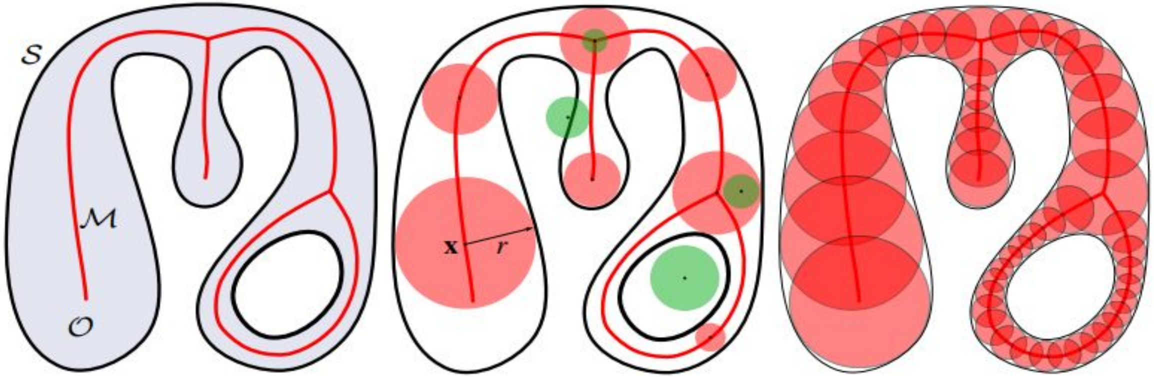

Figure 2: (a) The MAT skeleton M of the shape O with contour S. (b) Examples of maximally inscribed balls (red), a medial atom (x,r), and balls which are neither maximal nor inscribed, thus not contributing to M (green). (c) Approximate reconstruction of O by the union of balls B(x,r) given by a sparse sampling of M.

2. **Grassfire Analogy :** The Medial Axis Transform of O with boundary S is given by the shock graph of the motion $\dot{S}(t) = -n(t)$ and the time t when a shock is formed.

3. **Maxwell Set :** The Medial Axis Transform associates to a shape O the set of locations $M \in O$ with more than one corresponding closest point on the boundary S of O and their respective distances R to S.

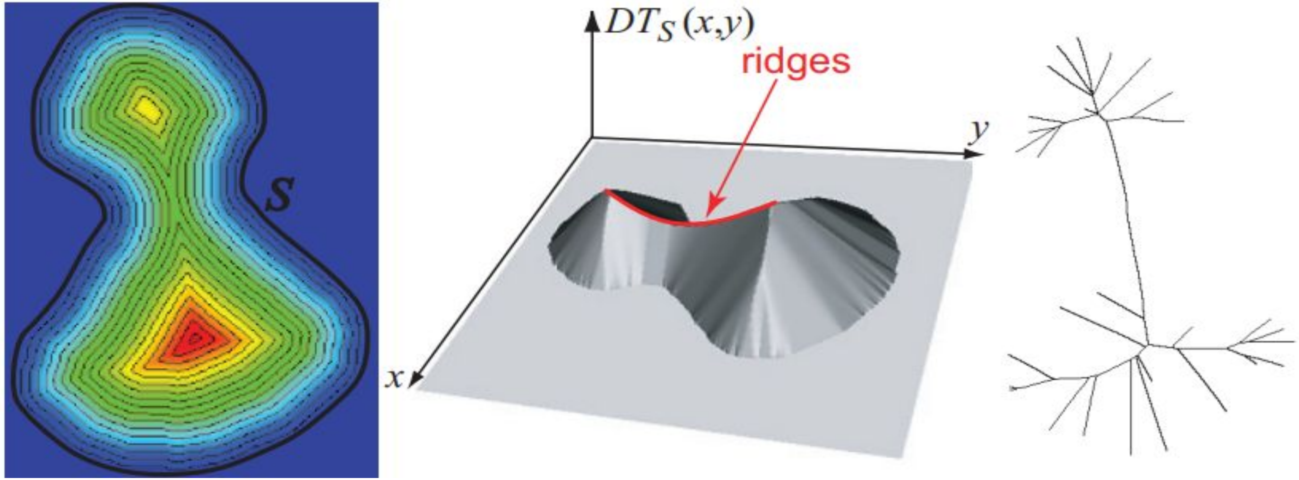

Figure 3: (a) 2D shape boundary S with its distance transform shown by color-coding and level sets. (b) Ridges of the distance transform plot. (c) Corresponding shape skeleton.

4. Symmetry Set : The Medial Axis Transform associates to O the set of centers M and radii R of all inscribed balls in O which are bi-tangent to its boundary S**.**

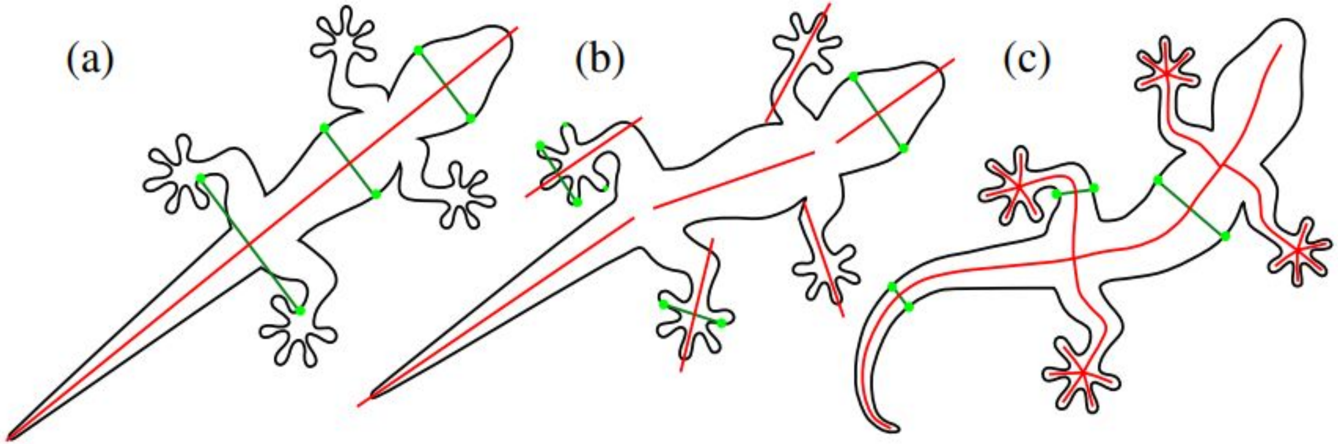

Figure 4: Three planar shapes Oa, Ob and Oc and their (a) global, (b) piecewise, and (c) local symmetry axes. Planar symmetry relations (green lines) between a few point-pairs (green dots) are highlighted. The medial axis encodes local reflectional symmetry.

Four alternative definitions of medial skeletons :

(a) Centers of maximally-inscribed balls
(b) Shock graph of the grassfire surface flow
(c) As points with more than one corresponding images on the surface
(d) Local axis of reflectional symmetry.

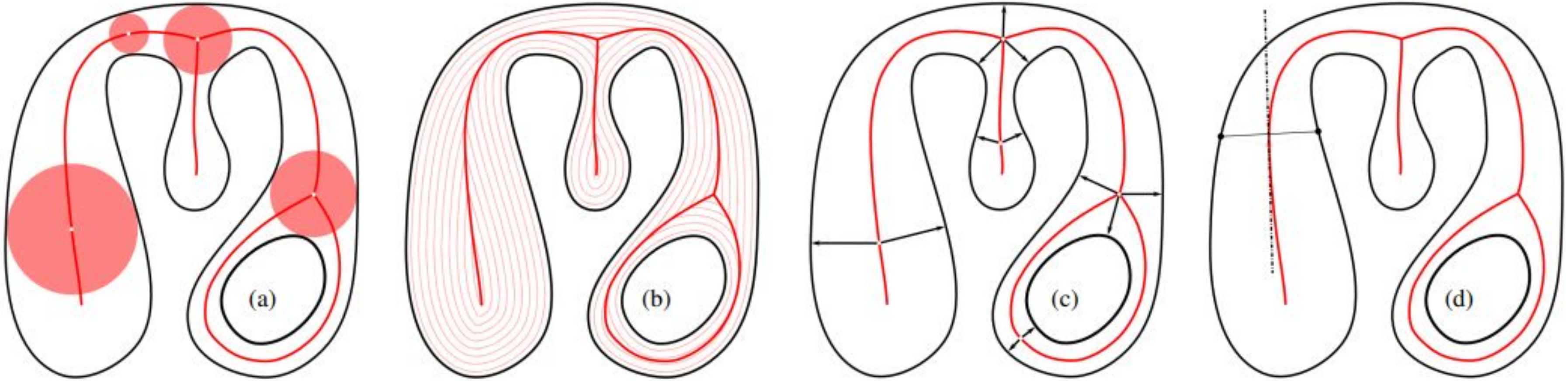

Figure 5: Four alternative definitions of medial skeletons: (a) centers of maximally-inscribed balls; (b) shock graph of the grassfire surface flow; (c) as points with more than one corresponding images on the surface; (d) local axis of reflectional symmetry.

Skeleton Properties : MAT can be seen as a dual representation which captures all shape aspects that a volumetric or boundary representation captures.

1. Topology Encoding
2. Skeletal Structure
3. Surface-Skeleton Correspondence
4. Semi-Continuity and instability

Properties of Skeletonization Method :

1. **Invariance :** Skeleton definition depends only on the shape O, and not on its position and/or size in the embedding space, skeletons should be invariant under isometric transforms T of the O, i.e., MAT(T(O)) = T(MAT(O)).

2. **Thinness :** Practical skeletons should be as thin as allowed by the space sampling used to model them. Mesh-based skeletons achieve the desired zero thickness by construction.

3. **Centeredness :** Each skeleton point should be at equal distance from at least two different points of the shape surface S. Voxel-based skeletons cannot always be perfectly centered, even for simple shapes. In contrast, mesh based surface skeletons can be computed with arbitrarily accurate centeredness.

4. Details Resolution : Skeletons should effectively and accurately capture all shape topology and geometry. This property is also known as the ability of skeletons to detect junctions or perform component-wise differentiation of the input shape parts, as well as detail preservation.

5. Regularization : Regularization can be seen as a filter $F(\cdot, \tau)$ which, when applied to MAT(O), yields a skeleton F(MAT(O)) that is Cauchy or Lipschitz continuous with respect to variations of O smaller than τ.

6. Scalability : Skeletons have been used in many applications. 2D skeletons require a relatively limited computing power, as typical input shapes are in the order of 1000^2 pixels. Voronoi-based methods typically achieve a complexity of O(nlogn) for n sample points on S.

Taxonomy of Skeleton :

1. **Surface Skeletons :** These methods aim to produce skeletons that follow the equivalent medial definitions. Such methods deliver skeletons which have a good correspondence to the input shape, and thereby a high reconstruction power.

a) **Analytic Surface Skeletons :** These methods represent both the input shape O and its surface skeleton analytically.

- Voronoi Method
- Bisector Method
- Shrinking Ball Method

b) **Image Surface Skeletons :** These methods use an image model of both the input shape O and the produced surface skeleton.

- Topological Thinning Method
- Distance Field Method

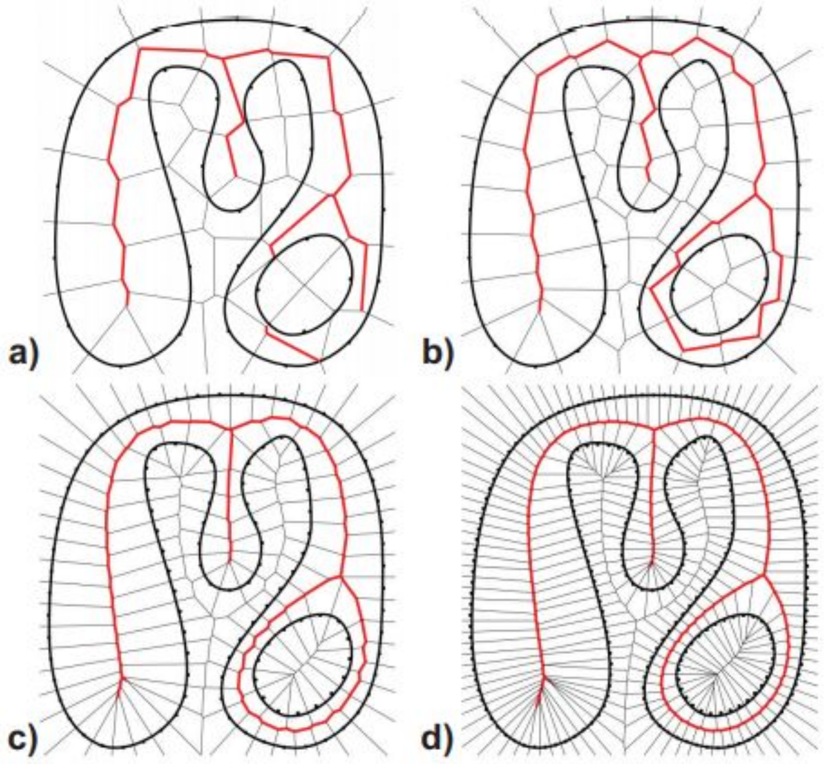

Figure 6: Voronoi diagram of a boundary with increasing and uniform sampling density. Voronoi vertices and edges completely enclosed in the boundary approximate the medial axis. As density increases, the approximation improves. A minimal sampling density is needed to obtain skeletons homotopic to the input shape.

2. Curve Skeletons : Curve skeletons are loosely defined as 1D structures “locally centered” in a shape.

a) **Analytic Curve Skeletons :** These methods represent both the shape O and its curve skeleton analytically.

- Medial-Surface-Based Methods
- Contraction Methods

b) **Image Curve Skeletons :** Image Curve Skeleton methods use an image model for both the input shape and the produced curve skeleton.

- Topological Thinning Methods
- Distance Based Methods

Reference :

- Tagliasacchi, Andrea, Thomas Delame, Michela Spagnuolo, Nina Amenta, and Alexandru Telea. "3d skeletons: A state-of-the-art report." In *Computer Graphics Forum*, vol. 35, no. 2, pp. 573-597. 2016.

Thank you